Kirchhoff’s Current Law

OBJECTIVES

■ Define Kirchhoff’s Current Law
■ Discuss how Kirchhoff’s Current Law applies to Series and Parallel Circuits
■ Calculate Current flow in Series and Parallel Circuits

INTRODUCTION

Kirchhoff’s Current Law is sometimes called “Kirchhoff’s First Law” or “Kirchhoff’s Junction Rule”, and along with Kirchhoff’s Voltage Law makes up the two fundamental laws of Electrical Engineering. In this lesson it will be shown how Kirchhoff’s Current Law describes the current flow through a junction of a circuit. This understanding will then be used to develope Kirchhoff’s Current Law applications in a Series and a Parallel circuit.

Kirchhoff’s Current Law

Kirchhoff’s Current Law helps to solve unknowns when working with electrical circuits. Kirchhoff’s Current Law with the addition of Kirchhoff’s Voltage Law and Ohm’s Law will allow for the solution of complex circuits.

Definition that will help in understanding Kirchhoff’s Current Law:

Junction - A junction is any point in a circuit where two or more circuit paths come together.

Examples of a Junction

Kirchhoff’s Current Law generally states:

The algebraic sum of all currents entering (+) and leaving (-) any point (junction) in a circuit must equal zero.

Restated as:

The sum of the currents into a junction is equal to the sum of the currents out of that junction.

In Figure 1, there is 3 Amps entering the junction and is split between two paths leaving the junction. The output current, 2 Amps for one path and 1 Amp for the other, has to equal that 3 Amps entering the junction.

Notice that the path for the measurement of the current is through the multimeter. The circuit has to be broken so that the current can go through the multimeter. (Review the Current Lab for further explanation.)
Kirchhoff’s Current Law expressed algebraically:

*The algebraic sum of all currents entering (+) and leaving (-) any point (junction) in a circuit must equal zero.*

\[ I(\text{in } 1) + I(\text{in } 2) + I(\text{in ...}) + I(\text{out } 1) + I(\text{out } 2) + I(\text{out ...}) = 0 \]

Example: 3 A + (-2 A) + (-1 A) = 0  
See Figure 1

The example above is the math equivalence for the junction in Figure 1. The negative sign indicates direction only (leaving a junction) and does not represent a negative current. This formula, although correct can be somewhat confusing. By rewriting the formula it can be made easier to understand.

Kirchhoff’s Current Law rewritten:

*The sum of the currents into a junction is equal to the sum of the currents out of that junction.*

\[ I(\text{in } 1) + I(\text{in } 2) + I(\text{in ...}) = I(\text{out } 1) + I(\text{out } 2) + I(\text{out ...}) \]

Example: 3 A = 2 A + 1 A  
See Figure 1

Converting to this equation removes the need to work with the negatives and for some people this equation is easier to remember and work with.

For a complete circuit this equation can be written using Current Total (It) on one side of the equation and all paths or branches on the other side of the equation.

**Application of Kirchhoff’s Current Law in a Series Circuit**

Remember that by definition a Series circuit has only one path for current. That means that the Total Current leaving the Voltage source has only one path back to the Voltage source. This further means that at any point along the path of a Series circuit the same amount of current will be flowing.

An equation that shows how Kirchhoff’s Current Law applies to a Series circuits is:

\[ \text{It} = I_{R_1} = I_{R_2} = I_{R_n} \ldots \]

$I_{R_1}$ stands for the Current (I) through R1. Likewise for the other Currents in the equation above.
The implications of this equation is that if you can solve for any Current (I, I_{R1}, I_{R2}, ...) in a series circuit you will know what the Current is for all of the circuit. This idea is sometimes stated as “Current is common in a series circuit.” See Figure 2.

![Figure 2](image2.png)

In Figure 2, notice that all of the multimeter measurement readings are the same and all are positive. The red probe of each multimeter is electrically closest to the positive terminal of the Voltage source, and the black probe is closest to the negative terminal of the Voltage source.

If the leads were reversed for any of the multimeters a negative reading would be made. This indicates that the Current is flowing through the meter in the opposite direction, this does not change the fact that the Current in the circuit is “common” and the formula stays the same. See Figure 3.

![Figure 3](image3.png)

Notice that the multimeter probes have been reversed and that the reading is now a negative.
Review for solving for Current in a Series Circuit

In an earlier lesson you will find 3 equations for solving for Current with Ohm’s/Watt’s Law.

One way to get the Total Current in the circuit above is to solve for the Total Resistance \( Rt \) using the equation \( Rt = R_1 + R_2 + \ldots \), which would give a value of 6 \( \Omega \) for the circuit above. Once the value for \( V_t \) (12 V) and the value for \( Rt \) (6 \( \Omega \)) is known, the equation \( V_t \div Rt = It \) can be used. \( 12 \text{ V} \div 6 \text{ \( \Omega \)} = 2 \text{ A} \). So in the circuit above the Total Current flowing through the circuit is 2 Amps.

Using Kirchhoff’s Current Law for a Series circuit \( It = I_{R_1} = I_{R_2} = I_R \ldots \) the current through all of the resistors are known. The Current through \( R_1 = It \) or 2 Amps, the Current through \( R_2 = It \) or 2 Amps, and finally the Current through \( R_3 = It \) or 2 Amps.

Is there another way to solve for the Currents in the above circuit? Yes, in the Kirchhoff’s Voltage Law lesson there was an equation to find the Voltage Drop for each known resistance in a series circuit.

Let’s use \( R_1 \) as an example, once the Voltage Drop is know for a resistance \( (V_{R_1}) \) and the resistance value is known \( R_1 \), then \( I_{R_1} \) can be calculated using \( V_{R_1} \div R_1 = I_{R_1} \). Remember \( I_{R_1} = It = I_{R_2} = I_{R_3} \ldots \)

\[
\frac{R_1}{Rt} \times V_s = V_{R_1}
\]

So,

\[
\frac{1 \text{ \( \Omega \)}}{6 \text{ \( \Omega \)}} \times 12 \text{ \( \text{V} \)} = 2 \text{ \( \text{V} \)}
\]

Solving for the Voltage Drop across \( R_1 \)
\[ \frac{V_{R1}}{R1} = I_{R1} \]

So,
\[ \frac{2 V}{1 \Omega} = 2 A \]

Solving for the Current through R1

Again, in a series circuit, if the current through one electrical component (Resistor in this circuit) is known, then the current through all the electrical components (Resistors in this circuit) is known.

**Application of Kirchhoff’s Current Law in a Parallel Circuit**

Remember that by definition a Parallel circuit has more than one path for current. That means that the Total Current leaving the Voltage source has more than one path back to the Voltage source, so the individual branches (paths) will have different Current going through them.

An equation that shows how Kirchhoff’s Current Law applies to a Parallel circuits is:

\[ I_t = I_{R1} + I_{R2} + I_{Rn} \ldots \]

The implication of this equation is that you have to solve for each Current (\( I_t, I_{R1}, I_{R2}, \ldots \)) individually in a parallel circuit. See Figure 4

![Figure 4](image)

Note that the Current going through R1 (3 A), R2 (4 A), and R3 (5 A) add up to the total Current leaving the Voltage Source B1 (12 A). Before it is shown how to calculate for these individual Currents, an understanding of the Current flowing through the circuit will be shown. Figure 5 shows the first branch (R1) of the above parallel circuit, 12 A enters the junction and is split into two paths, 3 A going to R1 and 9 A going toward R2 and R3. This complies with Kirchhoff’s Current Law, \( 12 = 9 + 3 \).

![Figure 5](image)
One way to get the Total Current for the circuit above is to solve for the Total Resistance (Rt) using the equation \( Rt = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \), which would give a value of 2 Ω. Once the value for \( V_t \) (24 V) and the value for \( Rt \) (2 Ω) is known, the equation \( V_t \div Rt = It \) can be used; 24 V \div 2 \Omega = 12 A. In the circuit above the Total Current flowing through the circuit is 12 Amps.

Another way to solve for the Total Current is to solve for all the individual branch Currents and then add them together. This is due to the additive nature of Kirchhoff’s Current Law for parallel circuits (\( It = I_{R_1} + I_{R_2} + I_{R_3} \ldots \)).

Figure 6 shows the second branch (R2) of the parallel circuit, 9 A enters the junction and is split into two paths, 4 A going to R2 and 5A going toward R3. This complies with Kirchhoff’s Current Law, 9 = 4 + 5.

Review for solving for Current in a Parallel Circuit

In an earlier lesson you will find 3 equations for solving for Current with Ohm’s/Watt’s Law.

If given a Parallel Circuit to solve for Total I (It):

One way to get the Total Current for the circuit above is to solve for the Total Resistance (Rt) using the equation \( Rt = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \), which would give a value of 2 Ω. Once the value for \( V_t \) (24 V) and the value for \( Rt \) (2 Ω) is known, the equation \( V_t \div Rt = It \) can be used; 24 V \div 2 \Omega = 12 A. In the circuit above the Total Current flowing through the circuit is 12 Amps.

Another way to solve for the Total Current is to solve for all the individual branch Currents and then add them together. This is due to the additive nature of Kirchhoff’s Current Law for parallel circuits (\( It = I_{R_1} + I_{R_2} + I_{R_3} \ldots \)).
Kirchhoff’s Voltage Law for a parallel circuit can help solve for Current. Kirchhoff’s Voltage Law for a parallel circuit indicates that the “Voltage is common” is a parallel circuit. \( (V_t = V_{R_1} = V_{R_2} = V_{R_3} \ldots) \).

See Figure 7.

Figure 7

To solve for any of the Currents going through the resistors in the parallel circuit in Figure 7, two electrical peremeters must be know. In this case Kirchhoff’s Voltage Law provided the Voltage across each resistor \( (V_{R_1}, V_{R_2}, \text{and} \ V_{R_3}) \). The Resistive values where given \( (R_1, R_2, \text{and} \ R_3) \). Using Ohm’s Law, the Current can be calculated using \( \frac{V}{R} = I \).

Solving for the Current through \( R_1 \): \[
\frac{V_{R_1}}{R_1} = I_{R_1} \rightarrow \frac{24 \ V}{4 \ \Omega} = 6 \ A
\]

Solving for the Current through \( R_2 \): \[
\frac{V_{R_2}}{R_2} = I_{R_2} \rightarrow \frac{24 \ V}{8 \ \Omega} = 3 \ A
\]

Solving for the Current through \( R_3 \): \[
\frac{V_{R_3}}{R_3} = I_{R_3} \rightarrow \frac{24 \ V}{8 \ \Omega} = 3 \ A
\]

Once the Current is known for all of the branches in a parallel circuit, the total current for the circuit can be found be adding all of the individual Currents together. Kirchhoff’s Current Law for a parallel circuit can be stated as \( I_t = I_{R_1} + I_{R_2} + I_{R_3} \ldots \).

Solving for the total current: \[
I_t = I_{R_1} + I_{R_2} + I_{R_3} \rightarrow 12 \ A = 6 \ A + 3 \ A + 3 \ A
\]

Practice Problems:
2.

\[ \begin{align*}
B1 & \quad 24 \text{ V} \\
\text{It} & = 2 \text{ A} \\
\end{align*} \]

\[ \begin{align*}
R1: 3 \Omega & \\
R2: 4 \Omega & \\
R3: & \\
\end{align*} \]

\[ \begin{align*}
V_{R1} & \\
V_{R2} & \\
V_{R3} & \\
I_{R1} & \\
I_{R2} & \\
I_{R3} & \\
\end{align*} \]

3.

\[ \begin{align*}
B1 & \quad 12 \text{ V} \\
\text{It} & = 2 \text{ A} \\
\end{align*} \]

\[ \begin{align*}
R1: 2 \Omega & \\
R2: 4 \Omega & \\
R3: 6 \Omega & \\
\end{align*} \]

\[ \begin{align*}
V_{R1} & \\
V_{R2} & \\
V_{R3} & \\
V_t & \\
I_{R1} & \\
I_{R2} & \\
I_{R3} & \\
\end{align*} \]

4.

\[ \begin{align*}
B1 & \quad 12 \text{ V} \\
\end{align*} \]

\[ \begin{align*}
R1: 2 \Omega & \\
R2: 4 \Omega & \\
R3: 6 \Omega & \\
\end{align*} \]

\[ \begin{align*}
V_{R1} & \\
V_{R2} & \\
V_{R3} & \\
I_{R1} & \\
I_{R2} & \\
I_{R3} & \\
I_t & \\
\end{align*} \]
5. 

\[
\begin{align*}
\text{B1} & \quad 36 \text{ V} \\
\text{R1} & \quad 12 \Omega \\
\text{R2} & \quad \text{V} \\
\text{R3} & \quad 18 \Omega \\
\end{align*}
\]

\(I_t = 6 \text{ A}\)

Extra Credit Problem:

6. 

\[
\begin{align*}
\text{B1} & \quad \text{V} \\
\text{R1} & \quad 2 \Omega \\
\text{R2} & \quad 4 \Omega \\
\text{R3} & \quad 6 \Omega \\
\end{align*}
\]

\(I_t = 18 \text{ A}\)

7. 

\[
\begin{align*}
\text{B1} & \quad 12 \text{ V} \\
\text{R1} & \quad 3 \Omega \\
\text{R2} & \quad 6 \Omega \\
\text{R3} & \quad 6 \Omega \\
\end{align*}
\]

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